K24U 2752

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/Improvement) Examination, November 2024
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
5B07MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Find the number of elements in the cyclic subgroup of $\mathbb{Z}_{30}$ generated by 25.

2. Give an example of an infinite group that is not cyclic.

3. Define alternating group.

4. Let $\phi : S_n \to \mathbb{Z}_2$ defined by $\phi(\sigma) = \begin{cases} 0, & \text{if } \sigma \text{ is an even permutation} \\ 1, & \text{if } \sigma \text{ is an odd permutation} \end{cases}$.
   Compute ker ϕ.

5. Describe all units in the ring $\mathbb{Q}$.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Prove that identity element in a group G is unique.

7. Describe all the elements in the cyclic subgroup of GL(2, $\mathbb{R}$) generated by $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

8. Find the number of generators of a cyclic group having order 60.

P.T.O.

9. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ be a permutation in S_6. Compute $|\langle \sigma \rangle|$.

10. Find all orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$ in S_6.

11. Prove that every group of prime order is cyclic.

12. Let ϕ be a homomorphism of a group G into a group G'. Then prove that
    i) if e is the identity element in G, then $\phi(e)$ is the identity element e' in G'.
    ii) if $a \in G$, then $\phi(a^{-1}) = (\phi(a))^{-1}$.

13. Let G be a group, and let $g \in G$. Let $\phi_g : G \to G$ be defined by $\phi_g(x) = gxg^{-1}$ for $x \in G$. Prove that ϕ_g is a homomorphism.

14. State the fundamental homomorphism theorem.

15. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.

16. Is every integral domain a field ? Justify.

PART – C

Answer **any 4** questions from this Part. **Each** question carries 4 marks **each**. **(4×4=16)**

17. Prove that a subset H of a group G is a subgroup of G if and only if
    i) H is closed under the binary operation of G.
    ii) the identity element e of G is in H.
    iii) for all $a \in H$ it is true that $a^{-1} \in H$ also.

18. State and prove division algorithm for $\mathbb{Z}$.

19. Find all subgroups of $\mathbb{Z}_{18}$ and draw the subgroup diagram for the subgroups.

20. Prove that every group is isomorphic to a group of permutations.

21. Let H be a subgroup of G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Prove that $\sim_L$ is an equivalence relation on G.

22. Show that an intersection of normal subgroups of a group G is again a normal subgroup of G.

23. Define subring. Let R be a ring and let a be a fixed element of R. Let $I_a = \{x \in R / ax = 0\}$. Show that I_a is a subring of R.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. a) Let G be a cyclic group with n elements generated by a. Let $b \in G$ and let $b = a^s$. Prove that

    i) b generates a cyclic subgroup H of G containing n/d elements, where d is the greatest common divisors of n and s.

    ii) $\langle a^s \rangle = \langle a^t \rangle$ if and only if gcd (s, n) = gcd (t, n).

    b) Find all generators of $\mathbb{Z}_{12}$.

25. a) List the elements in the dihedral group D_4.

    b) Find all subgroups of D_4 of order 2.

26. a) Let H be a normal subgroup of G. Prove that the cosets of H form a group G/H under the binary operation (aH)(bH) = (ab)H.

    b) Find the order of $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.

27. Prove that $F = \{a + b\sqrt{2} / a, b \in \mathbb{Q}\}$ with usual addition and multiplication forms a field.
